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BRST cohomology and nonlocal conserved charges

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ABSTRACT: A relation is found between nonlocal conserved charges in string theory and certain ghost-number two states in the BRST cohomology. This provides a simple proof that the nonlocal conserved charges for the superstring in an $AdS_5 \times S^5$ background are BRST-invariant in the pure spinor formalism and are κ-symmetric in the Green-Schwarz formalism.

KEYWORDS: AdS-CFT and dS-CFT Correspondence, BRST Symmetry.

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Contents

1. Introduction

Maldacena's conjecture that $d = 4$ $\mathcal{N} = 4$ super-Yang-Mills theory is dual to superstring theory in an $AdS_5 \times S^5$ background has been difficult to prove since the perturbative descriptions of these two theories do not overlap. To obtain non-perturbative information about the two theories, one possible tool could be integrability and there have been various papers discussing this possibility both on the super-Yang-Mills side and on the superstring side.

On the superstring side, Bena, Polchinski and Roiban [1] constructed an infinite set of nonlocal conserved charges for the Green-Schwarz (GS) superstring in an $AdS_5 \times S^5$ background,[1] suggesting an integrable structure. Vallilo [4] then constructed an analogous set of nonlocal conserved charges using the pure spinor formalism for the superstring in an $AdS_5 \times S^5$ background [5]. These nonlocal charges for the superstring were related in [6] to a corresponding set of nonlocal charges on the super-Yang-Mills side.

In the GS formalism for the superstring, invariance under κ-transformations is crucial for determining the physical spectrum. For example, classical κ-symmetry is preserved in a curved background when the background satisfies the low-energy supergravity equations of motion, so onshell massless vertex operators must be κ-symmetric at least at the classical level. Although quantum κ-transformations and massive GS vertex operators are not yet understood, it is reasonable to expect that all physical GS states should be κ-symmetric. This would imply that the conserved charges for physical symmetries should also be κ-symmetric.

In the pure spinor formalism for the superstring, the role of κ-symmetry is replaced by BRST invariance. In this case, quantization and massive vertex operators are well-understood, and the physical spectrum is described by states in the BRST cohomology. So physical symmetries in the pure spinor formalism must be BRST-invariant.

[1]These nonlocal charges were also independently found by Polyakov [2]. Similar nonlocal charges have been proposed in [3].

Surprisingly, it has not been previously verified if the nonlocal conserved charges of [1] are κ-symmetric, or if the nonlocal conserved charges of [4] are BRST-invariant. This led some people (including this author) to conclude that the charges were not κ-symmetric and to question their physical significance. Due to the insistence of Witten [7] that these nonlocal charges should describe physical symmetries in analogy with the nonlocal charges on the super-Yang-Mills side [6], the κ-symmetry and BRST invariance of the nonlocal charges of [1] and [4] were investigated.

As described in section 2, the existence of an infinite set of BRST-invariant nonlocal charges can be deduced from the absence of certain states in the BRST cohomology at ghost-number two. These ghost-number two states are $f_{AB}^{C} h^A h^B$ where h^A are the BRST-invariant ghost-number one states associated with the global isometries and f_{AB}^{C} are the structure constants. Whenever $f_{AB}^{C} h^A h^B$ can be written as $Q\Omega^C$ for some Ω^C (i.e. whenever $f_{AB}^{C} h^A h^B$ is not in the BRST cohomology), one can construct an infinite set of BRST-invariant nonlocal charges. It would be interesting to know if quantum corrections to the ghost-number two cohomology are related to the potential anomalies discussed in [8].

In section 3 of this paper, it is shown using the pure spinor formalism for the superstring in an $AdS_5 \times S^5$ background that the relevant ghost-number two states are absent from the classical BRST cohomology. The corresponding infinite set of BRST-invariant nonlocal charges is then explicitly constructed and shown to coincide with the conserved charges found by Vallilo in [4]. It is also shown that the conserved charges of Bena, Polchinski and Roiban in [1] are κ-symmetric.

2. Relation of nonlocal charges with BRST cohomology

Suppose one has a BRST-invariant string theory with global physical symmetries described by the charges $q^A = \int d\sigma\ j^A(\sigma)$. Since these symmetries take physical states to physical states, $q^A = \int d\sigma\ j^A(\sigma)$ must satisfy $Q(q^A) = 0$ where Q is the BRST charge. Note that $\{Q, b_0\} = H$ where H is the hamiltonian, so BRST invariance implies charge conservation if q^A commutes with the b_0 ghost, i.e. if q^A can be chosen in Siegel gauge. With the exception of the zero-momentum dilaton, it is expected that all ghost-number zero states in the BRST cohomology can be chosen in Siegel gauge.[2]

Since $Q(\int d\sigma j^A(\sigma)) = 0$, $Q(j^A) = \partial_\sigma h^A$ for some h^A of ghost-number one. And $Q^2 = 0$ implies that $Q(\partial_\sigma h^A) = 0$, which implies that $Q(h^A) = 0$ since there are no σ-independent worldsheet fields.

Consider the BRST-invariant ghost-number two states $f_{AB}^{C} : h^A h^B :$ where f_{AB}^{C} are the structure constants and normal-ordering is defined in any BRST-invariant manner, e.g. $: h^A(z) h^B(z) :\equiv \frac{1}{2\pi i} \oint dy (y-z)^{-1} h^A(y) h^B(z)$ where the contour of y goes around the point

[2] In the pure spinor formalism for the superstring, there is no natural b ghost. Nevertheless, it is expected that for any ghost-number zero state in the pure-spinor BRST cohomology, there exists a gauge in which the state is annihilated by H.

z. It will now be shown that whenever $f^C_{AB} : h^A h^B :$ is not in the BRST cohomology,[3] i.e. whenever there exists an operator Ω^C satisfying $Q(\Omega^C) = f^C_{AB} : h^A h^B :$, one can construct an infinite set of nonlocal BRST-invariant charges.

To prove this claim, consider the nonlocal charge

$$k^C = f^C_{AB} : \int_{-\infty}^{\infty} d\sigma \; j^A(\sigma) \; \int_{-\infty}^{\sigma} d\sigma' \; j^B(\sigma') : .$$

Using $Q(j^A) = \partial_\sigma h^A$, one finds that $Q(k^C) = \int d\sigma l^C(\sigma)$ where

$$l^C = -2 f^C_{AB} : h^A(\sigma) j^B(\sigma) : .$$

One can check that $Q(l^C) = f^C_{AB}\partial_\sigma(: h^A h^B :)$, so $Q(l^C - \partial_\sigma \Omega^C) = 0$ where Ω^C is the operator which is assumed to satisfy $Q(\Omega^C) = f^C_{AB} : h^A h^B :$.

Since $(l^C - \partial_\sigma \Omega^C)$ has $+1$ conformal weight and since BRST cohomology is only nontrivial at zero conformal weight, $l^C - \partial_\sigma \Omega^C = Q(\Sigma^C)$ for some Σ^C. Using Σ^C, one can therefore construct the nonlocal BRST-invariant charge

$$\widetilde{q}^C = f^C_{AB} : \int_{-\infty}^{\infty} d\sigma \; j^A(\sigma) \; \int_{-\infty}^{\sigma} d\sigma' \; j^B(\sigma') : - \int_{-\infty}^{\infty} d\sigma \Sigma^C(\sigma) \, .$$

By repeatedly commuting $\widetilde{q}^C$ with $\widetilde{q}^D$, one generates an infinite set of nonlocal BRST-invariant charges. So as claimed, $f^C_{AB} : h^A h^B := Q(\Omega^C)$ implies the existence of an infinite set of nonlocal BRST-invariant charges.

3. BRST-invariant charges in $AdS_5 \times S^5$ background

The results of the previous section will now be applied to the charges in an $AdS_5 \times S^5$ background using the pure spinor formalism for the superstring. As in the Metsaev-Tseytlin GS action in an $AdS_5 \times S^5$ background [10], the action in the pure spinor formalism [5] is constructed from left-invariant currents $J^A = (g^{-1}\partial g)^A$ where $g(x, \theta, \widehat{\theta})$ takes values in the coset $\mathrm{PSU}(2,2|4)/\,\mathrm{SO}(4,1) \times \mathrm{SO}(5)$, $A = ([ab], m, \alpha, \widehat{\alpha})$ ranges over the 30 bosonic and 32 fermionic elements in the Lie algebra of $\mathrm{PSU}(2,2|4)$, $[ab]$ labels the $\mathrm{SO}(4,1) \times \mathrm{SO}(5)$ "Lorentz" generators, $m = 0$ to 9 labels the "translation" generators, and $\alpha, \widehat{\alpha} = 1$ to 16 label the fermionic generators. The action in the pure spinor formalism also involves left and right-moving bosonic ghosts, $(\lambda^\alpha, w_\alpha)$ and $(\widehat{\lambda}^{\widehat{\alpha}}, \widehat{w}_{\widehat{\alpha}})$, which satisfy the pure spinor constraints $\lambda\gamma^m\lambda = \widehat{\lambda}\gamma^m\widehat{\lambda} = 0$. These pure spinor ghosts transform as spinors under the local $\mathrm{SO}(4,1) \times \mathrm{SO}(5)$ transformations and couple to the $AdS_5 \times S^5$ spin connection in the worldsheet action through their Lorentz currents $N_{ab} = \frac{1}{2} w\gamma_{ab}\lambda$ and $\widehat{N}_{ab} = \frac{1}{2}\widehat{w}\gamma_{ab}\widehat{\lambda}$.

[3] This BRST cohomology is defined in the "extended" Hilbert space which includes the zero mode of the x^m variables. As explained in [9], the inclusion of the x^m zero mode in the Hilbert space allows global isometries to be described by ghost-number one states in the cohomology.

To construct the nonlocal charges, the notation and conventions of [4] will be used where

$$
\begin{aligned}
J_0 &= \left(g^{-1}\partial g\right)^{[ab]} T_{[ab]}\,, & J_1 &= \left(g^{-1}\partial g\right)^{\alpha} T_{\alpha}\,, & J_2 &= \left(g^{-1}\partial g\right)^{m} T_{m}\,,\\
J_3 &= (g^{-1}\partial g)^{\widehat{\alpha}} T_{\widehat{\alpha}}\,, & N &= \frac{1}{2}\left(w\gamma^{[ab]}\lambda\right) T_{[ab]} & &\\
\overline{J}_0 &= \left(g^{-1}\overline{\partial} g\right)^{[ab]} T_{[ab]}\,, & \overline{J}_1 &= \left(g^{-1}\overline{\partial} g\right)^{\alpha} T_{\alpha}\,, & \overline{J}_2 &= \left(g^{-1}\overline{\partial} g\right)^{m} T_{m}\,,\\
\overline{J}_3 &= \left(g^{-1}\overline{\partial} g\right)^{\widehat{\alpha}} T_{\widehat{\alpha}}\,, & \widehat{N} &= \frac{1}{2}\left(\widehat{w}\gamma^{[ab]}\widehat{\lambda}\right) T_{[ab]}\,, & &\\
\partial &= \frac{1}{2}\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\sigma}\right), & \overline{\partial} &= \frac{1}{2}\left(\frac{\partial}{\partial\tau}-\frac{\partial}{\partial\sigma}\right), & &
\end{aligned}
\tag{3.1}
$$

and T_A are the PSU$(2,2|4)$ Lie algebra generators. It will be convenient to also introduce the notation

$$\lambda = \lambda^\alpha T_\alpha, \quad \widehat{\lambda} = \widehat{\lambda}^{\widehat{\alpha}} T_{\widehat{\alpha}}\,.$$

Note that λ and $\widehat{\lambda}$ are fermionic since $(T_\alpha, T_{\widehat{\alpha}})$ are fermionic and $(\lambda^\alpha, \widehat{\lambda}^{\widehat{\alpha}})$ are bosonic.

Under classical BRST transformations generated by

$$Q = \int d\sigma(\lambda^\alpha J_3^{\widehat{\alpha}} + \widehat{\lambda}^{\widehat{\alpha}}\overline{J}_1^{\alpha})\delta_{\alpha\widehat{\alpha}}\,,$$

g transforms by right-multiplication as

$$Q(g) = g(\lambda + \widehat{\lambda}) \tag{3.2}$$

and the pure spinors transform as

$$Q(N) = -2[J_3, \lambda]\,, \qquad Q(\widehat{N}) = -2[\overline{J}_1, \widehat{\lambda}]\,, \qquad Q(\lambda) = Q(\widehat{\lambda}) = 0\,.$$

The left-invariant currents therefore transform as

$$
\begin{aligned}
Q(J_j) &= \delta_{j+3,0}\partial\lambda + [J_{j+3}, \lambda] + \delta_{j+1,0}\partial\widehat{\lambda} + \left[J_{j+1}, \widehat{\lambda}\right],\\
Q(\overline{J}_j) &= \delta_{j+3,0}\overline{\partial}\lambda + \left[\overline{J}_{j+3}, \lambda\right] + \delta_{j+1,0}\overline{\partial}\widehat{\lambda} + \left[\overline{J}_{j+1}, \widehat{\lambda}\right],
\end{aligned}
$$

where j is defined modulo 4, i.e. $J_j \equiv J_{j+4}$.

To prove the existence of an infinite set of BRST-invariant charges, one needs to find an $\Omega = \Omega^C T_C$ satisfying $Q\Omega =: \{h, h\}$: where $h = h^A T_A$, $Q(j) = \partial_\sigma h$, and $q^A = \int d\sigma j^A$ are the charges associated with the global PSU$(2,2|4)$ isometries. It will be shown at the end of subsection 3.1 that $Q(j) = \frac{1}{2}\partial_\sigma(g(\lambda - \widehat{\lambda})g^{-1})$, so

$$h = \frac{1}{2} g(\lambda - \widehat{\lambda})g^{-1}\,. \tag{3.3}$$

Note that h is BRST-invariant since

$$Q(h) = \frac{1}{2} g\{(\lambda + \widehat{\lambda}), (\lambda - \widehat{\lambda})\}g^{-1} = \frac{1}{2} g((\lambda^\alpha \gamma^m_{\alpha\beta}\lambda^\beta)T_m - (\widehat{\lambda}^{\widehat{\alpha}}\gamma^m_{\widehat{\alpha}\widehat{\beta}}\widehat{\lambda}^{\widehat{\beta}})T_m)g^{-1} = 0 \tag{3.4}$$

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because of the pure spinor constraint. Consider the ghost-number two state

$$\{h,h\} = \frac{1}{2}g(\lambda-\widehat{\lambda})(\lambda-\widehat{\lambda})g^{-1} = -\frac{1}{2}g\{\lambda,\widehat{\lambda}\}g^{-1}\,. \tag{3.5}$$

Since $\{\lambda+\widehat{\lambda},\lambda+\widehat{\lambda}\} = 2\{\lambda,\widehat{\lambda}\}$, one can write this state as $Q\Omega$ where

$$\Omega = -\frac{1}{4}g(\lambda+\widehat{\lambda})g^{-1}\,. \tag{3.6}$$

So $Q\Omega = \{h,h\}$, which implies the existence of an infinite set of BRST-invariant charges.

3.1 Explicit construction of BRST-invariant nonlocal charges

To explicitly construct these BRST-invariant charges, suppose one has a current whose τ-component a satisfies

$$Qa = \partial_\sigma\Lambda + [a,\Lambda] \tag{3.7}$$

for some Λ. Then the charge

$$\begin{aligned} P(e^{-\int_{-\infty}^{\infty}d\sigma\ a(\sigma)}) \equiv 1 - \int_{-\infty}^{\infty}d\sigma a(\sigma) + \int_{-\infty}^{\infty}d\sigma a(\sigma)\int_{-\infty}^{\sigma}d\sigma' a(\sigma') - \\ - \int_{-\infty}^{\infty}d\sigma a(\sigma)\int_{-\infty}^{\sigma}d\sigma' a(\sigma')\int_{-\infty}^{\sigma'}d\sigma'' a(\sigma'') + \cdots \end{aligned} \tag{3.8}$$

satisfies $Q(P(e^{-\int_{-\infty}^{\infty}d\sigma\ a(\sigma)})) = 0$. So $P(e^{-\int_{-\infty}^{\infty}d\sigma\ a(\sigma)})$ is a BRST-invariant charge.

To construct a satisfying (3.7), consider

$$a(c_j,\overline{c}_j) = g(c_0N + c_1J_1 + c_2J_2 + c_3J_3 + \overline{c}_0\widehat{N} + \overline{c}_1\overline{J}_1 + \overline{c}_2\overline{J}_2 + \overline{c}_3\overline{J}_3)g^{-1}\,, \tag{3.9}$$

where c_j and $\overline{c}_j$ are constant coefficients. Note that $a(c_j,\overline{c}_j)$ is invariant under the local $\mathrm{SO}(4,1)\times\mathrm{SO}(5)$ transformations.

Using the BRST transformations of (3.2),

$$\begin{aligned} Qa = {}& g\left[\lambda+\widehat{\lambda}, c_0N + \overline{c}_0\widehat{N} + \sum_{k=1}^{3}(c_kJ_k + \overline{c}_k\overline{J}_k)\right]g^{-1} + \\ &+ g\left(-2c_0[J_3,\lambda] - 2\overline{c}_0[J_1,\widehat{\lambda}] + c_1\partial\lambda + c_3\partial\widehat{\lambda} + \overline{c}_1\overline{\partial}\lambda + \overline{c}_3\overline{\partial}\widehat{\lambda}\right)g^{-1} + \\ &+ g\sum_{k=1}^{3}\left(c_k[J_{k+3},\lambda] + c_k[J_{k+1},\widehat{\lambda}] + \overline{c}_k[\overline{J}_{k+3},\lambda] + \overline{c}_k[\overline{J}_{k+1},\widehat{\lambda}]\right)g^{-1}\,. \end{aligned} \tag{3.10}$$

And defining

$$\Lambda(b,\overline{b}) = g(b\lambda + \overline{b}\widehat{\lambda})g^{-1}\,,$$

where b and $\overline{b}$ are constant coefficients, one obtains

$$\begin{aligned} \partial_\sigma\Lambda + [a,\Lambda] = {}& g\left(b(\partial\lambda - \overline{\partial}\lambda) + \overline{b}(\partial\widehat{\lambda} - \overline{\partial}\widehat{\lambda})\right)g^{-1} + g\left[\sum_{j=0}^{3}(J_j - \overline{J}_j), b\lambda + \overline{b}\widehat{\lambda}\right]g^{-1} + \\ &+ g\left[c_0N + \overline{c}_0\widehat{N} + \sum_{k=1}^{3}(c_kJ_k + \overline{c}_k\overline{J}_k), b\lambda + \overline{b}\widehat{\lambda}\right]g^{-1}\,. \end{aligned} \tag{3.11}$$

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Setting (3.10) equal to (3.11) and using the worldsheet equations of motion [5, 4]

$$\overline{\partial}\lambda + [\overline{J}_0, \lambda] = -\frac{1}{2}[\widehat{N}, \lambda]\,, \qquad \partial\widehat{\lambda} + [J_0, \widehat{\lambda}] = -\frac{1}{2}[N, \widehat{\lambda}]\,, \tag{3.12}$$

and the pure spinor constraints $[\lambda, N] = [\widehat{\lambda}, \widehat{N}] = 0$, one obtains the conditions

$$\begin{array}{cccc}
c_1 = b\,, & -\overline{c}_3 = \overline{b}\,, & & \\
-c_1 + c_2 = bc_1 + b\,, & -c_2 + c_3 = bc_2 + b\,, & -c_3 - 2c_0 = bc_3 + b\,, & \\
-c_1 = \overline{b}c_1 + \overline{b}\,, & -c_2 + c_1 = \overline{b}c_2 + \overline{b}\,, & -c_3 + c_2 = \overline{b}c_3 + \overline{b}\,, & 2c_0 + c_3 = -2\overline{b}c_0 + \overline{b}\,, \\
2\overline{c}_0 + \overline{c}_1 = -2b\overline{c}_0 - b\,, & -\overline{c}_1 + \overline{c}_2 = b\overline{c}_1 - b\,, & -\overline{c}_2 + \overline{c}_3 = b\overline{c}_2 - b\,, & -\overline{c}_3 = b\overline{c}_3 - b\,, \\
-\overline{c}_1 - 2\overline{c}_0 = \overline{b}\overline{c}_1 - \overline{b}\,, & -\overline{c}_2 + \overline{c}_1 = \overline{b}\overline{c}_2 - \overline{b}\,, & -\overline{c}_3 + \overline{c}_2 = \overline{b}\overline{c}_3 - \overline{b}\,. &
\end{array} \tag{3.13}$$

The conditions of (3.13) are solved by

$$\begin{array}{llll}
c_0 = \frac{1}{2}\left(1 - \mu^2\right), & c_1 = \pm\mu^{1/2} - 1\,, & c_2 = \mu - 1\,, & c_3 = \pm\mu^{3/2} - 1\,, \\
\overline{c}_0 = \frac{1}{2}\left(\mu^{-2} - 1\right), & \overline{c}_1 = 1 \mp \mu^{-3/2}\,, & \overline{c}_2 = 1 - \mu^{-1}\,, & \overline{c}_3 = 1 \mp \mu^{-1/2}\,, \\
b = \pm\mu^{1/2} - 1\,, & \overline{b} = \pm\mu^{-1/2} - 1\,, & &
\end{array} \tag{3.14}$$

which coincides with the solution of [4] for conserved currents.

Note that the global charge $q = \int d\sigma j(\sigma)$ can be obtained from (3.9) by expanding $a(\mu)$ near $\mu = 1$. If $\mu = 1 + \epsilon$, one finds that $a(\mu) = \epsilon j + \mathcal{O}(\epsilon^2)$ and $\Lambda(\mu) = \epsilon h + \mathcal{O}(\epsilon^2)$ where $Q(j) = h$. So from the formulas

$$b(\mu) = \mu^{1/2} - 1 = \frac{1}{2}\epsilon + \mathcal{O}(\epsilon^2)\,, \qquad \overline{b}(\mu) = \mu^{-1/2} - 1 = -\frac{1}{2}\epsilon + \mathcal{O}(\epsilon^2)\,,$$

one learns that

$$h = \lim_{\epsilon\to 0} \epsilon^{-1}\Lambda(\mu) = \frac{1}{2}g(\lambda - \widehat{\lambda})g^{-1}\,, \tag{3.15}$$

as was claimed in (3.3).

3.2 κ-symmetry of nonlocal GS charges

Finally, it will be shown that the nonlocal GS conserved charges of [1] are κ-symmetric. In conformal gauge for the GS superstring, the κ-transformations of g and the κ-transformations of the left-invariant currents can be obtained from the BRST transformations of (3.2) by replacing λ with $[\kappa, J_2]$ and replacing $\widehat{\lambda}$ with $[\widehat{\kappa}, \overline{J}_2]$ where $\kappa = \kappa^{\widehat{\alpha}}T_{\widehat{\alpha}}$ and $\widehat{\kappa} = \widehat{\kappa}^{\alpha}T_{\alpha}$. This is the $AdS_5 \times S^5$ version of the procedure adopted in [11] where λ^{α} is replaced by $\Pi^m(\gamma_m\kappa)^{\alpha}$ and $\widehat{\lambda}^{\widehat{\alpha}}$ is replaced by $\overline{\Pi}^m(\gamma_m\widehat{\kappa})^{\widehat{\alpha}}$. In GS conformal gauge, $\eta_{mn}J_2^m J_2^n = \eta_{mn}\overline{J}_2^m\overline{J}_2^n = 0$ and the κ-transformations are constrained to satisfy $\kappa^{\widehat{\alpha}}\overline{J}_1^{\alpha}\delta_{\alpha\widehat{\alpha}} = \widehat{\kappa}^{\alpha}J_3^{\widehat{\alpha}}\delta_{\alpha\widehat{\alpha}} = 0$ so that the h_{zz} and $h_{\overline{z}\overline{z}}$ components of the worldsheet metric do not transform. Together with the GS equations of motion $[J_2, \overline{J}_1] = 0$ and $[\overline{J}_2, J_3] = 0$, these conditions imply that

$$[\lambda, J_2] = [\widehat{\lambda}, \overline{J}_2] = [\lambda, \overline{J}_1] = [\widehat{\lambda}, J_3] = 0\,. \tag{3.16}$$

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Using the current of (3.9) with $c_0 = \overline{c}_0 = 0$, one finds that (3.10) is equal to (3.11) if

$$\begin{aligned}
c_1 &= b\,, & -\overline{c}_3 &= \overline{b}\,, & & \\
-c_1 + c_2 &= bc_1 + b\,, & -c_3 &= bc_3 + b\,, & & \\
-c_1 &= \overline{b}c_1 + \overline{b}\,, & -c_2 + c_1 &= \overline{b}c_2 + \overline{b}\,, & c_3 &= \overline{b}\,, \\
\overline{c}_1 &= -b\,, & -\overline{c}_2 + \overline{c}_3 &= b\overline{c}_2 - b\,, & -\overline{c}_3 &= b\overline{c}_3 - b\,, \\
-\overline{c}_1 &= \overline{b}\overline{c}_1 - \overline{b}\,, & -\overline{c}_3 + \overline{c}_2 &= \overline{b}\overline{c}_3 - \overline{b}\,. & &
\end{aligned} \tag{3.17}$$

The conditions of (3.17) are solved by

$$\begin{aligned}
c_1 &= \pm\mu^{1/2} - 1\,, & c_2 &= \mu - 1\,, & c_3 &= \pm\mu^{-1/2} - 1 \\
\overline{c}_1 &= 1 \mp \mu^{1/2}\,, & \overline{c}_2 &= 1 - \mu^{-1}\,, & \overline{c}_3 &= 1 \mp \mu^{-1/2}\,, \\
b &= \pm\mu^{1/2} - 1\,, & \overline{b} &= \pm\mu^{-1/2} - 1\,, & &
\end{aligned} \tag{3.18}$$

which coincides with the conserved GS charges of [1].

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